Theoretical Limitations to Ferromagnetic Parametric Amplifier Performance*

R. W. DAMON† AND J. R. ESHBACH†

Summary—It has been commonly expected that improved operation of the ferrite parametric amplifier could be obtained by use of materials of narrower resonance linewidth, ΔH. This parameter is critical in determining the pumping power (P_p) required for operation of the device. Also of importance, however, is the limitation of device properties determined by the dependence on ΔH of the instability threshold of the spin-wave system. Considering this limitation, the maximum voltage gain-fractional bandwidth product ($g_v\Delta\omega/\omega_1$) has been determined as a function of other device parameters, and typical values calculated for several modes of operation. In the electromagnetic mode, for example, there is an optimum ΔH which yields maximum $g_v\Delta\omega/\omega_1$ at a given pumping power. It is also shown that a minimum filling factor, also a function of ΔH for some types of operation, is required to reach the oscillation threshold even in the unloaded device.

Introduction

THE parametric amplifier is based on a regenerative feedback process which results from coupling together two tuned circuits with a time-varying reactance. For particular conditions of tuning, and for a sufficiently large modulation amplitude of the variable reactance, the input impedance of the tuned circuits exhibits a negative resistance, which can be used to obtain amplification of applied signals or to produce oscillations at the natural frequencies of the tuned circuits.

One of the earliest proposals for a parametric amplifier at microwave frequencies utilized the ferromagnetic resonance effect, the uniform precession of the magnetization serving as a means of coupling between the tuned circuits.[1] This proposal was based on earlier work[2] which led to an interpretation of some anomalous results obtained in resonance experiments at high microwave power levels.[3,4] These anomalous results were shown to arise from inherent instabilities in the uniform motion of the magnetization. At sufficiently high power levels, these instabilities prevent further increase in the amplitude of the uniform mode and couple energy from the uniform precession into short wavelength (spin-wave) modes of motion of the sample magnetization. The effect is similar to the processes used in parametric amplification, but the modes into which the energy is dissipated are unsuitable for device use because the short wavelength of these modes makes it impossible to couple out useful energy. Instead the existence of these instabilities can interfere with the operation of a parametric amplifier if energy from the precessing uniform magnetization is dissipated in these modes at a pump power-level less than that required for operation of the amplifier.

The pump frequency magnetic field strength required for a specified amplifier performance can be calculated, as can the critical RF field strength for the onset of these instabilities. The purpose of this note is to point out the limitations imposed on amplifier design by the existence of the potentially unstable behavior of the magnetization. The results will be expressed as the maximum gain-bandwidth product attainable for the device. Since the results depend on the properties of both the sample and of the circuits to which it is coupled, only typical values can be given, but similar calculations can be performed for any specific device following the method outlined here.

General Procedure

The principles of the analysis can be illustrated by the diagram shown as Fig. 1. The uniform mode of precession is represented by the magnetization vector M precessing about the steady magnetic field H, driven by the circularly polarized pumping magnetic field of amplitude h. At the resonance frequency, the angle θ between M and H is approximately $\theta = h/\Delta H$, where ΔH is the width of the resonance line. The coupling between signal and idler circuits increases with increasing θ, so a large precession angle is desired to obtain good amplifier performance.

The angle θ_{th} represents the threshold angle for operation of a parametric device. At θ_{th}, the transverse magnetization couples sufficient energy to the two pertinent cavity modes to overcome the losses of the cavities. As an amplifier, the gain becomes unity for $\theta = \theta_{th}$. The coupling is proportional to the total transverse magnetization, so θ_{th} depends on the size of the sample, or the filling factor, as well as on the losses of the resonant systems.

While θ_{th} is the minimum angle for operation of a parametric amplifier, an increased gain-bandwidth product is obtained by coupling an external load and increasing the precession angle. The maximum gain-bandwidth product increases roughly as $(\theta/\theta_{th})^2$, and

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† G.E. Res. Lab., Schenectady, N. Y.

[1] H. Suhl, "The theory of the ferromagnetic microwave amplifier," *J. Appl. Phys.*, vol. 28, pp. 1225–1236; November, 1957.

[2] H. Suhl, "The theory of ferromagnetic resonance at high signal powers," *J. Phys. Chem., Solids*, vol. 1, pp. 209–227; 1957.

[3] R. W. Damon, "Relaxation effects in the ferromagnetic resonance," *Rev. Mod. Phys.*, vol. 25, pp. 239–245; January, 1953.

[4] N. Bloembergen and S. Wang, "Relaxation effects in para- and ferromagnetic resonance," *Phys. Rev.*, vol. 93, pp. 72–83; January, 1954.

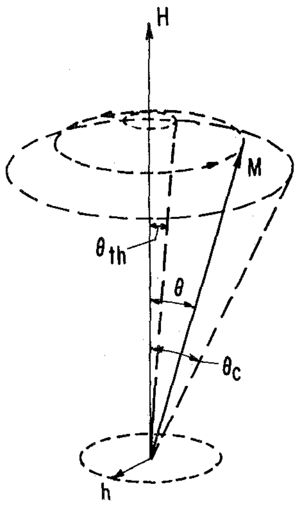

Fig. 1—Vector diagram representing uniform precession of magnetization M about the uniform magnetic field H, driven by the circularly polarized pumping field h. At resonance, M and h rotate about H at the Larmor frequency, with M at an angle $\theta = h/\Delta H$ with H. In parametric amplifier use, θ_{th} is the threshold angle for net gain. The critical angle θ_c is the limit determined by spin-wave instability.

since θ varies inversely with ΔH, a narrow linewidth material is desirable to obtain large θ at a given pump power. The variation of θ with linewidth is shown in Fig. 2 for several values of pump magnetic field strength.

The angle θ_c represents the critical angle for spin-wave instability. The value of θ_c depends only on properties of the magnetic material, and represents the angle at which the transverse magnetization is large enough to couple energy catastrophically from the uniform precession into certain spin-wave modes. For angles $\theta < \theta_c$, the coupling to the spin-waves acts as a damping mechanism on the motion of M, but at $\theta = \theta_c$ the spin-wave system absorbs all additional energy fed into the uniform mode and any attempt to increase θ only increases the oscillation amplitude of the spin-wave mode while leaving $\theta = \theta_c$. The critical angle, θ_c, is determined by the losses of the pertinent spin-wave mode, and is independent of sample size.

It should be noted that the steady-state precession angle for the uniform mode has been calculated only for the case of degeneracy of the uniform mode and the subsidiary absorption which arises from spin-waves of half the pump frequency.[2] In assuming that the precession angle cannot exceed θ_c in the nondegenerate case, we are utilizing experimental results obtained on Ni ferrite which show a power dependence in agreement with this assumption.[3] Such a well-defined critical angle may not be found in all materials. In fact, Schlömann[5] has recently extended Suhl's calculations to the nondegenerate case considered here, and shows that in magnetically inhomogeneous materials, the power coupled to the spin-waves increases smoothly with θ and that no well-defined critical angle exists. Recent experiments by Green verify this result in some materials.[6] For these materials, the results obtained in our analysis would be modified. Since no well-defined critical angle exists, there is no absolute limit to amplifier properties, but only a practical limit arising from increased loss, and decreased pump efficiency, as the precession angle is increased.

A combination of the homogeneous and inhomogeneous mechanisms is likely, exhibiting a decline in pump efficiency at the critical angle, but with the possibility of a slow increase in precession angle beyond θ, with further increase in pump power. The present results then would correspond to the practical limit of device properties.

In one important case,[7] the relation between critical angle and loss is given by $\theta_c = \sqrt{2\Delta H_k / 4\pi M}$, where ΔH_k represents the losses in the spin-wave modes of interest. The maximum precession angle, and thus the maximum amplifier performance, is limited by the inherent instability of the spin-wave system. Since it has been found that ΔH_k may be considerably less than ΔH in some materials,[8] we will retain the distinction in most of our mathematical results. In other materials, however, it appears approximately correct to use $\Delta H_k = \Delta H$, and this will be assumed in numerical examples. The observed linewidth, ΔH, represents the sum of all loss mechanisms, so ΔH_k cannot exceed ΔH. Thus this latter assumption leads to the largest value of θ_c for a given linewidth. The limiting precession angle is shown in Fig. 2 for this case.

This figure shows that if a given pumping power is available, there is an optimum linewidth material giving maximum θ. If the linewidth is too large, the pump cannot supply enough power to reach a large precession angle. If too narrow a linewidth is used, the precession angle is limited by instability effects. When the pump field available is $h = 1$ oe, for example, $\Delta H = 9.5$ oe provides the maximum value of θ for sample magnetization of $4\pi M = 1700$ gauss. If $\Delta H < 9.5$ oe, θ is limited to a θ_c which is less than the optimum and the pump power is not utilized. If $\Delta H > 9.5$ oe, θ is limited by the material losses and a larger pump power is needed to achieve the same amplifier characteristisc.

The optimum linewidth is that for which $\theta = \theta_c$ at the available pumping field. If $\Delta H_k = \Delta H$, this condition re-

[5] E. Schlömann, "Ferromagnetic resonance at high signal powers," *Bull. Am. Phys. Soc.*, Ser. II, vol. 4, p. 53; January, 1959.

[6] J. J. Green and E. Schlömann, "High Power Ferromagnetic Resonance at X-Band in Polycrystalline Garnets," PGMTT NATIONAL SYMPOSIUM, Cambridge, Mass.; June 1–3, 1959.

[7] We assume that the subsidiary mode, arising from spin-waves of half the pump frequency, is not degenerate with the uniform mode, since this degeneracy would lead to an exceptionally small value of θ_c. Thus we assume $f_p > 2N_T(\gamma/2\pi) \cdot 4\pi M$, where f_p is pump frequency. N_T is the transverse demagnetizing factor of the sample and $\gamma/2\pi = 2.8$ mc/gauss. For spherical samples, $N_T = \frac{1}{3}$, and this condition requires $f_p > 3400$ mc for yttrium iron garnet and $f_p > 8000$ mc for manganese ferrite. With disks magnetized normally, $N_T \rightarrow 0$, and nondegenerate behavior is obtained at any operating frequency.

[8] R. C. LeCraw and E. G. Spencer, "Surface-independent spin-wave relaxation in ferromagnetic resonance of yttrium iron garnet," *J. Appl. Phys.*, vol. 30, pp. 185S–186S; April, 1959.

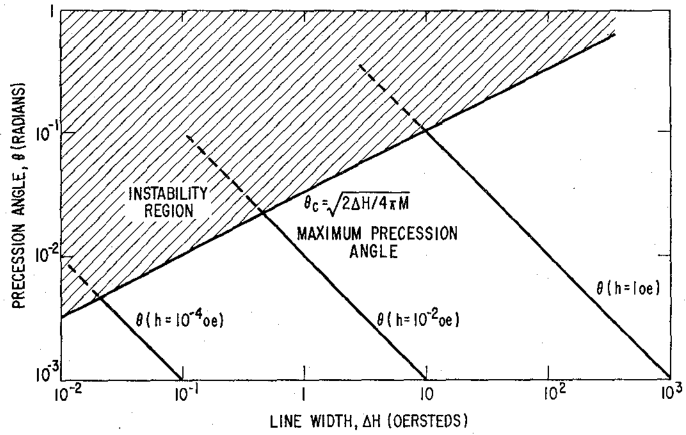

Fig. 2—For a given pump field strength h, material of narrower line width ΔH permits attainment of a larger precession angle θ. The critical angle for spin-wave instability, θ_c, is also a function of linewidth and sets an upper limit to θ for a given pump field strength.

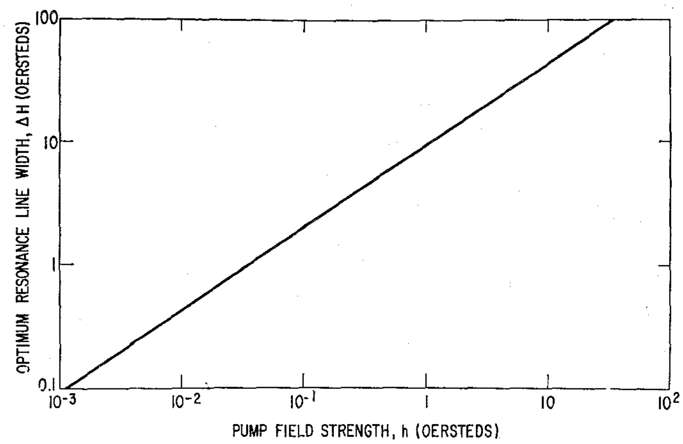

Fig. 3—At a given value of pump field strength, the optimum linewidth for parametric amplifier use is determined by the spin-wave instability limit, and is shown in this graph of $(\Delta H)_{\text{opt}}$ vs h.

sults in the optimum linewidth

$$\Delta H_{\text{opt}} = (\tfrac{1}{2}h^2 \cdot 4\pi M)^{1/3} \tag{1}$$

This is plotted in Fig. 3 for $4\pi M = 1700$ gauss. This expression for ΔH enables us to calculate the maximum gain-bandwidth product in terms of available pump power for typical values of θ_{th} in each of several modes of amplifier operation.

Application to Specific Amplifiers

Electromagnetic Operation

Suhl has calculated the gain-bandwidth product for this mode of operation,[1] obtaining, for large gains

$$g_v \Delta\omega/\omega_1 = \frac{1}{\beta Q_1}\left[(\theta/\theta\text{th})^2 - 1\right]. \tag{2}$$

Here g_v is the voltage gain at the signal frequency ω_1, $\Delta\omega/\omega_1$ is the fractional bandwidth, Q_1 is the unloaded Q of the signal cavity and β is a factor of order unity. For optimum performance, the operating conditions are chosen so $\theta = \theta_c = \sqrt{2\Delta H_k/4\pi M}$.

In determining θ_{th}, we restrict the analysis to spherical samples to avoid complicating demagnetizing effects. The results for other shapes would differ by only small factors. In a cavity having only the fields h_{z_1} and h_{x_2} at the signal and idler frequencies respectively,

$$\theta_{th} = \frac{2\omega_1/\gamma}{4\pi M} \frac{1}{F\sqrt{Q_1 Q_2}},$$

where the filling factor is defined as

$$F = \int_{\text{sample}} h_{z_1} h_{x_2} dv \Big/ \left[\int_{\text{cavity}} h_{z_1}{}^2 dv \int_{\text{cavity}} h_{x_2}{}^2 dv\right]^{1/2}.$$

In this mode of operation, the threshold angle is independent of sample linewidth, and can be represented as shown in Fig. 4 for typical cavity and sample parameters and for several values of F. Clearly if θ_{th} exceeds θ_c, operation of the amplifier is impossible, and the ratio of

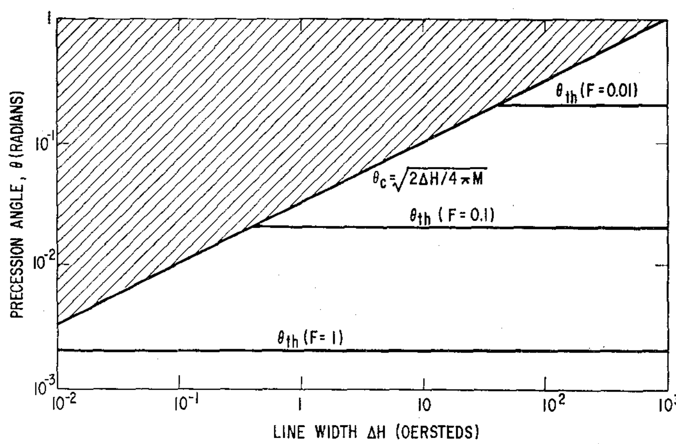

Fig. 4—In electromagnetic operation, the threshold precession angle for net gain is a function of frequency, circuit losses, sample magnetization and filling factor. It is independent of linewidth. For three values of filling factor F, threshold angles are shown with the other parameters: $4\pi M = 1700$, $\omega_1{}^2 = 10^{21}$, and $Q_1 = Q_2 = 10^3$. Since the maximum precession angle θ_c is a function of linewidth, ΔH, the amplifier properties will also be a function of ΔH.

θ_c to θ_{th} at the linewidth appropriate to the sample used determines the maximum gain-bandwidth product of the device. The important point is that the threshold angle is determined by the cavity configuration, while the spin-wave instability limit decreases for narrow linewidth, leading to a reduction in the maximum gain-bandwidth product when narrow linewidth materials are used in this mode of operation.

A quantitative statement of this relation is obtained by substituting in (2) the expression for θ_{th} and using $\theta = \theta_c$, giving

$$(g_v \Delta\omega/\omega_1)_{\max} = \frac{1}{\beta Q_1}\left[\frac{F^2 Q_1 Q_2 \cdot 4\pi M}{2\omega_1{}^2/\gamma^2} \Delta H_k - 1\right]. \tag{3}$$

From (3) it is clear that materials are desired for which the spin-wave linewidth is large. But since $\Delta H_k \leq \Delta H$, and ΔH should be small to permit large θ at the given pump level, it is desirable to obtain $\Delta H_k = \Delta H$. This gives the maximum value of $g_v \Delta\omega/\omega_1$ for a given pump power level. The desirability of a large filling factor is also evident, and in fact, no operation is obtained unless $F^2 > (2\omega_1^2/\gamma^2)/4\pi M Q_1 Q_2 \Delta H$. Thus, although narrow linewidth materials reduce pump power requirements, an increased filling factor is required to achieve a given gain-bandwidth product.

It is instructive to calculate values of $g_v \Delta\omega/\omega_1$ for typical device parameters. We have chosen the signal frequency to be $f_1 \approx 5000$ mc ($\omega^2{}_1 = 10^{21}$ sec^{-2}), and have taken the unloaded Q of both signal and idle circuit as 10^3. Since a pump level of a few watts or less is desirable, the optimum linewidth is under 20 oe. Only yttrium iron garnet has linewidths in this range, so the value $4\pi M = 1700$ gauss has been used. Substituting in (3) we find $(g_v \Delta\omega/\omega_1)_{max} = 10^{-3}[340F^2\Delta H - 1]$. This is plotted in Fig. 5 for several values of F. A linewidth of 3 oe, for example, would be a desirable value since this requires a pump field strength of only 0.2 oe. With some care a filling factor of 0.1 should be possible. The maximum performance attainable under these conditions corresponds to 20-db gain and 5 mc/sec bandwidth at the signal frequency of 5000 mc/sec.

Only one device has been reported using the electromagnetic mode of operation,[9] and in this case a manganese ferrite sample was employed. Assuming the sample had a linewidth of 50 oe, the optimum pump field-strength is $h = \Delta H\sqrt{2\Delta H/4\pi M} \sim 8$ oe. For the conditions under which the device was operated, this was probably achieved. The theoretical gain-bandwidth product for the frequency, Q, and sample magnetization used is $g_v \Delta\omega/\omega_1 = 2\times 10^{-3}\ (10^4 F^2 - 1)$. The observed value of 4×10^{-3} implies $F = 1.7\times 10^{-2}$. Calculations of F using a reasonable field configuration and the known sample size lead to values in agreement with this. Note that a value of at least $F = 1\times 10^{-2}$ is required for even marginal operation.

Semistatic Operation

For a given sample size, improved filling factor and increased gain-bandwidth product can be obtained by using a magnetostatic mode of the sample[10] as one of the resonant systems required in the parametric amplifier. The theoretical gain-bandwidth product has not been reported for this case, but can be readily calculated in direct analogy to the electromagnetic mode. Eq. (2) is obtained with an appropriate redefinition of θ_{th}.

If the signal circuit is a cavity mode with unloaded loss characterized by Q_1, and the idle mesh is a magnetostatic mode of linewidth ΔH_2, then the threshold angle is

$$\theta_{th} = \frac{1}{F}\sqrt{\frac{\Delta H_2}{4\pi M}}\,\frac{1}{\sqrt{Q_1}}.$$

[9] M. T. Weiss, "Solid-state microwave amplifier and oscillator using ferrite," *J. Appl. Phys.*, vol. 29, p. 421; March, 1958.

[10] R. L. White and I. H. Solt, Jr., "Multiple ferromagnetic resonance in ferrite spheres," *Phys. Rev.*, vol. 104, pp. 56–62; October 1, 1956. Also L. R. Walker, "Magnetostatic modes in ferromagnetic resonance," *Phys. Rev.*, vol. 105, pp. 390–399; January 15, 1957.

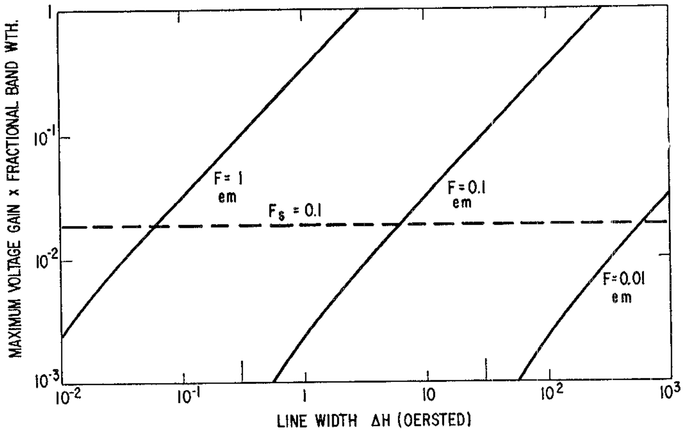

Fig. 5—For the parameters used in Fig. 4 the maximum voltage-gain bandwidth product is calculated as a function of linewidth. The solid lines are for electromagnetic operation, and the dashed line corresponds to semistatic operation.

The filling factor is

$$F = \frac{1}{2\sqrt{2}}\,\frac{\left|\int_s V_{z1}(M_{x2} + iM_{y2})dv\right|}{\left[\mathrm{Im}\int_s M_{x2}M_{y2}{}^* dv\right]^{1/2}\left[\int_c V_1{}^2 dv\right]^{1/2}},$$

where the M's are magnetization components of the magnetostatic mode and the V's are the vector potential functions of the cavity mode.[1] In contrast to the electromagnetic case, θ_{th} decreases with linewidth in the same way as θ_c, since one resonant mode of the amplifier is a magnetostatic mode. Typical values of θ_{th} are shown in Fig. 6. If $\Delta H_2 \sim \Delta H_k$, the required filling factor for semistatic operation and the maximum gain-bandwidth product are independent of linewidth. Material of narrow linewidth is thus desired to minimize the pump power required, being limited only by the necessity of having the resonance sufficiently wide to meet bandwidth specifications.

The maximum gain-bandwidth product can be calculated from the expression

$$g_v \Delta\omega/\omega_1 = \frac{1}{\beta Q_1}\,[2F^2 Q_1 \Delta H_k/\Delta H_2 - 1].$$

If $\Delta H_2 = \Delta H_k$, it is seen that $F > 1/\sqrt{2Q_1}$ is required for operation, and

$$g_v \Delta\omega/\omega_1 = \frac{1}{\beta Q_1}\,(2F^2 Q_1 - 1).$$

This expression is plotted in Fig. 5 for typical values of Q and F.

The semistatic type of operation offers the advantage of large filling factor, even for samples of moderate size. While in electromagnetic operation, the filling factor $F_{em} \approx$ (sample volume per cavity volume), it is found for simple field configurations that semistatic operation leads to $F_{ss} \approx$ (sample volume per cavity volume)$^{1/2}$. Thus with limited size samples, substantially improved performance should be obtained in the latter case. If

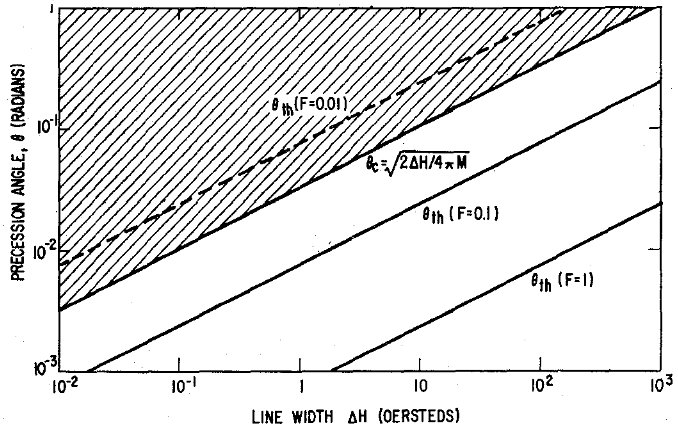

Fig. 6—In semistatic operation, the threshold precession angle for net gain is a function of losses in the signal circuit and in the magnetostatic mode used as idler, and of sample magnetization and filling factor. The dependence on magnetostatic mode losses, or linewidth, leads to amplifier gain-bandwidth independent of ΔH. For several values of filling factor F, θ_{th} is shown with other parameters $\Delta H_2 = \Delta H$, $4\pi M = 1700$, $Q_1 = 10^3$. The dashed line for $F = 0.01$ indicates that net gain cannot be observed, since θ cannot achieve the threshold value.

sample dimensions are taken similar to those used in achieving $F_{em} = 0.1$ in the electromagnetic case, a value $F_{ss} = 0.3$ will be obtained in semistatic operation. This would permit substantial improvement in gain bandwidth product, to a value of 10^3 mc at a signal frequency of 5000 mc/sec, corresponding to about 100 mc bandwidth at 20-db gain if ΔH_2 does not limit the bandwidth.

Operation of this type of amplifier has also been reported by Weiss.[9] He gives only incomplete data, but it is interesting to compare these results with the present limitations. The pump power of 40 watts corresponds to a microwave field strength of about $\frac{1}{2}$ oe in his structure. In turn, this is the critical field for a YIG sample ($4\pi M = 1700$ gauss) having the reasonable linewidth $\Delta H = 6$ oe. To obtain device operation, we require $\theta_{th} < h_p/\Delta H$. For the assumed parameters, this can be achieved if $F > 3.1 \times 10^{-2}$. This value is in approximate agreement with the filling factor estimated for Weiss's device. That such a large filling factor can be obtained with a sample volume one-sixteenth that which Weiss used in electromagnetic operation is indeed a tribute to the semistatic device.

Magnetostatic Operation

In magnetostatic operation, no microwave cavity resonance is necessary; two magnetostatic modes of the sample are used as the two parametric amplifier meshes. Still further improvement in filling factor is achieved for a given sample volume. Suhl has pointed out the limitations imposed by the fact that the magnetostatic mode spectrum could provide alternative pairs of magnetostatic modes which might break into oscillation and preclude operation of this type of device; we consider the limitations set by spin-wave instabilities even if these alternative modes can be avoided.

The lowest frequency for magnetostatic modes is $\omega_{\min} = \gamma(H - N_z \cdot 4\pi M)$, while in cylindrically symmetric samples the resonant frequency at which it is desired to pump is

$$\omega_p = \gamma \left[H + \frac{1 - 3N_z}{2} 4\pi M \right].$$

To obtain magnetostatic operation it is necessary to have $\omega_{\min} \leq \frac{1}{2}\omega_p$. This condition imposes the restriction on pump frequency $\omega_p \leq \gamma(1 - N_z) 4\pi M$. But this is also the condition for the occurrence of a particularly low instability threshold. This decreased instability threshold arises from a coincidence of the main resonance and a subsidiary absorption by a set of spinwaves driven at twice their natural frequency. From the above result, the conditions for coincidence will be satisfied whenever magnetostatic operation is possible. In this case the critical angle for spin-wave instability is $\theta_c = \alpha \Delta H_k / 4\pi M$, where α is a factor of order unity which depends on sample magnetization, frequency and geometry.

The threshold angle for operation in the magnetostatic mode has been previously derived[1] as

$$\theta_{th} = \frac{1}{F_{ms}} \sqrt{\frac{\Delta H_1}{4\pi M}} \sqrt{\frac{\Delta H_2}{4\pi M}} \, .$$

The filling factor, F_{ms}, has been calculated by Suhl[1] and will not be repeated here. It is a measure of the overlap of the two magnetostatic modes relative to the magnetic energy of these modes. Combining θ_{th} with the maximum precession angle, θ_c, the maximum gain-bandwidth product is found,

$$(g_v \Delta\omega/\omega_1)_{\max} = \frac{\gamma \Delta H_1}{\beta Q_1} \left[\frac{(\alpha \Delta H_k)^2}{\Delta H_1 \Delta H_2} F^2 - 1 \right].$$

Since F is necessarily less than unity, no operation is obtained unless $\Delta H_1 \; \Delta H_2 / (\alpha \Delta H_k)^2 \leq 1$. Thus in addition to the potential instabilities arising from unwanted pairs of magnetostatic modes, the excitation of spinwave modes restricts the performance of the magnetostatic type of amplifier. Since the linewidth of the signal mode, ΔH_1, must include the coupled load, the performance of this class of amplifier is seriously impaired.

Modified Semistatic Operation

A fourth mode of operation has been proposed by Berk, Kleinman and Nelson,[11] and experimentally studied by Whirry and Wang.[12] As in the semistatic type, a sample mode is used as idler. This device differs from the earlier proposal, however, since it utilizes the uniform mode for this function rather than a higher

[11] A. D. Berk, L. Kleinman and C. E. Nelson, "Modified semistatic ferrite amplifier," 1958 WESCON CONVENTION RECORD, pt. 3, pp. 9–12.

[12] W. L. Whirry and F. B. Wang, "Experimental Study of the Modified Semistatic Ferrite Amplifier," presented at Conference on Magnetism and Magnetic Materials, Philadelphia, Pa.; November 17–20, 1958.

order magnetostatic mode. The pump is necessarily at a higher frequency and does not have the advantage of driving the sample at resonance. The resultant decrease in pump effectiveness is partially compensated by the possibility of obtaining higher cavity Q at the pump frequency.

The threshold pump field strength has been calculated by Berk, *et al.* To utilize our expression for gain-bandwidth product, it is desirable to convert this to a threshold precession angle. Since the pump is not at the resonance frequency, the angle is obtained by dividing h_{th} by the difference between pump frequency and resonance frequency, $\theta_{th}=\gamma h_{th}/(\omega_p-\omega_r)=\gamma h_{th}/\omega_1$. The result obtained is

$$\theta_{th}=\frac{1}{F}\left[\frac{\Delta H}{4\pi M}\frac{1}{Q_1}\right]^{1/2},$$

where

$$F=\left[\int_s h_1^2dv \Big/ \int_c h_1^2dv\right]^{1/2}$$

is the filling factor for the signal mode.[13] θ_{th}, as shown in Fig. 6 for semistatic operation, is also applicable to this type of operation.

Since the coupling from the precessing magnetization driven at the pump frequency to spin-waves of the same frequency depends only on the precession angle, the critical angle for spin-wave instability is unchanged from our previous value, $\theta_c=\sqrt{2\Delta H_k/4\pi M}$. Coupling this with the threshold angles we find the maximum gain-bandwidth product

$$(g_v\Delta\omega/\omega_1)_{\max}=\frac{1}{Q_1\beta}\left[F^2Q_1\frac{2\Delta H_k}{\Delta H}-1\right].$$

If $\Delta H_k=\Delta H$, this expression is independent of linewidth as in the conventional semistatic case, since the decreased loss in the uniform mode used as idler reduces the threshold angle at the same rate that the critical angle is decreased. This device thus should achieve a gain-bandwidth product comparable to that in semistatic operation.

Other Devices

The preceding analysis has been limited to the regenerative type of amplifier, utilizing cavity or sample modes as resonant circuits. Distributed amplifiers probably offer advantages of larger bandwidth or increased stability, but it should be noted that similar limitations on the maximum precession angle are imposed by the instabilities considered here.[14] The nonlinear nature of the uniform mode of the magnetization has also been applied in the construction of microwave frequency converters and multipliers. It has not previously been pointed out, however, that the maximum output of these devices will also be limited by the spin-wave instabilities to a level corresponding to the critical RF field-strength of the local oscillator, or pump source.

Conclusion

It has been shown that inherent instabilities in the uniform motion of the magnetization limit the performance of the ferrite parametric amplifier. A general expression for the voltage gain-bandwidth product has been utilized and applied to four different types of amplifiers. From the threshold precision angle for device operation and the critical angle for spin-wave instability, it is found that a minimum filling factor is required for a given ΔH, regardless of pump power, to obtain even marginal operation of any of these devices. It is also noted that a material linewidth should be chosen which permits the precession angle to be opened out to the critical angle at the available pump power level. For filling factors exceeding the minimum requirements, the maximum gain-bandwidth product possible has been calculated. The results are summarized in Table I.

TABLE I

Mode of Operation	Maximum Voltage Gain Fractional Bandwidth
Electromagnetic	$1/\beta Q_1\left[\frac{F^2Q_1Q_2 4\pi M}{2\omega_1^2/\gamma^2}\Delta H_k-1\right]$
Semistatic	$1/\beta Q_1\left[2F^2Q_1\frac{\Delta H_k}{\Delta H_2}-1\right]$
Magnetostatic	$1/\beta Q_1\left[2F\frac{(\alpha\Delta H_k)^2}{\Delta H_1\Delta H_2}-1\right]$
Modified Semistatic	$1/\beta Q_1\left[2F^2Q_1\frac{\Delta H_k}{\Delta H}-1\right]$

The gain-bandwidth product possible in practical devices is substantial, in spite of this limitation, provided that fairly large filling factors can be obtained. The limits set by this analysis, however, together with previous expressions for pumping power requirements, serve as a useful basis for comparing the ferrite amplifier with other types of amplifiers.

[13] The usage of the symbol F is slightly different from Berk, *et al.*,[11] in order to conform to comparable terminology used in the remainder of this paper.

[14] P. K. Tien and H. Suhl, "A traveling-wave ferromagnetic amplifier," Proc. IRE, vol. 46, pp. 700–706; April, 1958.